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The effect of magnetic field on the ground and excited states of the two-dimensional D^- centre

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Abstract

The ground and excited state energies of the two-dimensional D^- centre have been calculated, respectively, as a function of magnetic field. The critical magnetic field values at which the excited states change from unbound to bound have been found. The optical transition between two bound states is discussed. Numerical results with the hyperspherical approach are in good agreement with those obtained through other intensive numerical methods and those obtained through experiments.

1. Introduction

There has been increasing interest, both experimental and theoretical, in the investigation of two-dimensional (2D) and quasi-two-dimensional (Q2D) systems such as quantum wells and superlattices because of their intrinsic physical interest and their technological applications in electronic devices [1–3]. In multiple-quantum-well (MQW) structures, intentional doping may induce the formation of D^- states. Selective doping of GaAs/GaAlAs quantum wells makes it possible to realize the situation in which some of the electrons weakly bound to shallow donors in the GaAlAs barrier are transferred to the GaAs quantum well. In the quantum well they are trapped by neutral donors D^0 forming there a stable population of negatively charged donor centres D^- containing two electrons. Due to the existence of electron–electron (e–e) interaction it would be expected that, compared to D^0, some new phenomena would appear in D^-. In addition, a D^0 centre is a simple two-body system, while a D^- centre is a three-body one, thus the exact solution of the D^- centre problem would help us to understand the e–e correlation effects in low dimensional systems.

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Simple D^- centres have been unambiguously identified in bulk samples of GaAs by means of the magnetic-field dependence of their photoionization threshold [4]. Experiments on the magneto-optical transition associated with D^- centres in GaAs/GaAlAs quantum wells [2] revealed the effect of magnetic field on the ground and excited states of these centres [3].

The D^- states are always discrete because of the Landau quantization of electron motion in the well planes and the subband quantization of motion along the magnetic field [3]. Only some of these discrete states are bound, in the sense that their energy lies lower than the energy of a donor plus the energy of a free electron infinitely far from the donor and in the lowest Landau level. The binding energy of a D^- centre can be expressed by

$$E_b = E_{D^0} + E_e - E_{D^-}$$

where, if $E_b > 0\ (< 0)$, the D^- states are the bound (unbound) states. The binding energy of a D^- centre in the magnetic field has been calculated with different models [3, 5, 6]. However, the results obtained are in the limit of either a low or a high magnetic field. For example, the ground-state binding energy of a D^- centre in a low magnetic field was obtained by a variational method [5], the precision of which is particularly dependent upon the variational wave function selected, and the calculation of the excited states is questionable. The energy of four bound states were obtained in the high magnetic field limit [3, 6], but their variation as a function of magnetic field is unknown. Up to now, no clear picture has been given for the variation of binding energy with magnetic field. In this article, we introduce a hyperspherical approach to study the energy of a D^- centre and discuss the behaviour of its binding energy in a continuously changing magnetic field. We also obtain the critical magnetic field so that we can know its optical transition spectrum characteristics.

2. Theory

The Schrödinger equation for a D^- centre in an external magnetic field $\vec{B} = \nabla \times \vec{A}$, which is applied perpendicular to the quantum well interface, is:

$$\left(\frac{1}{2m^*}\sum_{i=1}^{2}\left(\vec{p}_i + \frac{e}{c}\vec{A}\right)^2 + \sum_{i<j}^{3}\frac{Z_i Z_j \mathrm{e}^2}{\epsilon r_{ij}}\right)\psi\left(\vec{r}_1, \vec{r}_2\right) = E\psi\left(\vec{r}_1, \vec{r}_2\right) \tag{1}$$

where $\vec{r}_1$ and $\vec{r}_2$, respectively, are the position vectors of electrons 1 and 2 in the $x - y$ plane, ϵ denotes the static dielectric constant of the material, and Z_i is the sign of the charge of the ith particle.

To solve equation (1) in two-dimensional space, the hyperspherical coordinates (HC)$\{R, \phi, \varphi_1, \varphi_2\}$ are introduced:

$$\left[-\frac{1}{2}\left(\frac{\partial^2}{\partial R^2} + \frac{3}{R}\frac{\partial}{\partial R} - \frac{\Lambda^2(\Omega)}{R^2}\right) + \frac{U(\Omega)}{R} + \frac{1}{8}\gamma^2 R^2 + \frac{1}{2}\gamma\left(-\mathrm{i}\frac{\partial}{\partial\varphi_1} - \mathrm{i}\frac{\partial}{\partial\varphi_2}\right)\right]\psi(R, \Omega) = E\psi(R, \Omega) \tag{2}$$

where

$$\Lambda^2(\Omega) = -\left[\frac{\partial^2}{\partial\phi^2} + \left(\frac{\cos\phi}{\sin\phi} - \frac{\sin\phi}{\cos\phi}\right)\frac{\partial}{\partial\phi} - \frac{\hat{l}^2(\varphi_1)}{\cos^2\phi} - \frac{\hat{l}^2(\varphi_2)}{\sin^2\phi}\right] \tag{3}$$

is the grand orbital operator, $\hat{l}(\varphi_i) = -\mathrm{i}\partial/\partial\varphi_i$, and

$$U(\Omega) = \left(\frac{1}{2}\right)^{\frac{1}{2}}\frac{1}{\cos\phi^{(a)}} - \frac{1}{\cos\phi^{(b)}} - \frac{1}{\cos\phi^{(c)}} \tag{4}$$

where Ω is a symbol denoting the angular variables $\{\phi, \varphi_1, \varphi_2\}$ compactly. The definitions of R, ϕ, φ_1 and φ_2 can be found in [7–11]. The three different sets of relative coordinates are schematically shown in figure 1. Correspondingly, the mass-weighted relative coordinates $\{\vec{\eta}_1^{(j)}, \vec{\eta}_2^{(j)}\}$, $(j = a, b, c)$ are defined by

$$\begin{aligned}
&\vec{\eta}_1^{(a)} = \left(\frac{1}{2}\right)^{\frac{1}{2}} (\vec{r}_2 - \vec{r}_1)\,, \vec{\eta}_2^{(a)} = \sqrt{2}\left(\vec{r}_3 - \frac{\vec{r}_1 + \vec{r}_2}{2}\right);\\
&\vec{\eta}_1^{(b)} = \vec{r}_3 - \vec{r}_2, \vec{\eta}_2^{(b)} = \vec{r}_1 - \vec{r}_3;\\
&\vec{\eta}_1^{(c)} = \vec{r}_1 - \vec{r}_3, \vec{\eta}_2^{(c)} = \vec{r}_2 - \vec{r}_3;
\end{aligned} \tag{5}$$

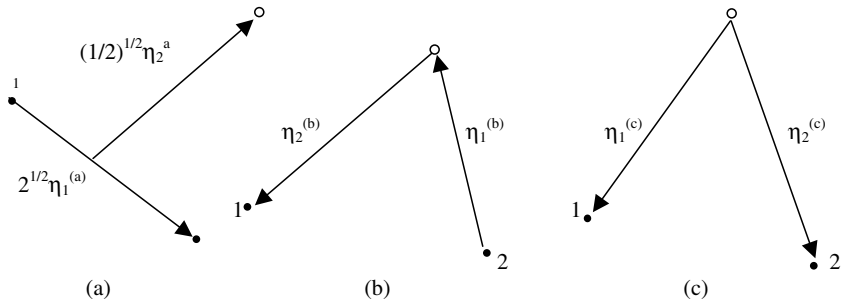

Figure 1. Mass-weighted relative coordinates for a D^- centre. In set (a), $\sqrt{2}\vec{\eta}_1^{(a)}$ denotes the displacement from electron 1 to electron 2, $\sqrt{\frac{1}{2}}\vec{\eta}_2^{(a)}$ denotes the displacement from the centre of mass of two electrons to the impurity state. In set (b), $\vec{\eta}_1^{(b)}$ denotes the displacement from electron 2 to the impurity state, $\vec{\eta}_2^{(b)}$ denotes the displacement from the impurity state to electron 1. In set (c), $\vec{\eta}_1^{(c)}$ and $\vec{\eta}_2^{(c)}$ denote the displacements from the impurity state to electrons 1 and 2, respectively.

where $\vec{r}_3$ is the position vector of the impurity state. The mass of the impurity state is infinite and $\vec{r}_3 \to 0$. In equation (2) and in the following the unit of energy is $R_y^* = \mathrm{e}^4 m_e^*/\epsilon^2\hbar^2$, while the units for length and magnetic field are $a_0^* = \epsilon\hbar^2/m_e^*\mathrm{e}^2$ and $\gamma = e\hbar B/m_e^* c R_y^*$, respectively.

In the coordinate set (a), the common eigenfunctions of the operators $\{\Lambda^2(\Omega), \hat{l}(\varphi_1), \hat{l}(\varphi_2)\}$, known as the hyperspherical harmonics $Y_{\{\nu,l_1,l_2\}}(\Omega)$, are given by

$$Y_{\{\nu,l_1,l_2\}}(\Omega) = \Xi_\nu^{l_1 l_2} P_\nu^{l_1 l_2}(\phi)\mathrm{e}^{\mathrm{i}l_1\varphi_1}\mathrm{e}^{\mathrm{i}l_2\varphi_2} \tag{6}$$

with separate quantum numbers λ, l_1 and l_2, while $\lambda = 2\nu + |l_1| + |l_2|$. In equation (6), $\Xi_\nu^{l_1 l_2}$ is a normalization constant, $P_\nu^{l_1 l_2}(\phi)$ is a Jacobi polynomial, and their analytical expressions and the eigenequations that $Y_{\{\nu,l_1,l_2\}}(\Omega)$ satisfy can be seen in [11]. The quantum number l_1 is odd for spin triplet states (space antisymmetric wave functions) whereas l_1 is even for spin singlet states (space symmetric wave functions) due to the electron exchange symmetry. However, in coordinate sets (b) or (c), l_1 can be even or odd. Accordingly, the hyperspherical harmonics in the set (c) are defined as

$$K_{[\lambda]}^{\pm}(\Omega) = \begin{cases} \frac{1}{\sqrt{2}}\left(Y_{\{\nu,l_1,l_2\}}(\Omega) \pm Y_{\{\nu,l_2,l_1\}}(\Omega)\right) & (l_1 \neq l_2) \\ Y_{\{\nu,l_1,l_2\}}(\Omega) & (l_1 = l_2) \end{cases} \tag{7}$$

where the + and − superscripts refer to space symmetric and space antisymmetric states, respectively. For compactness we use a symbol $[\lambda]$ to denote the full set of quantum numbers $\{\nu, l_1, l_2\}$. The Zeeman spin energy of electrons is neglected here.

Having obtained the hyperspherical harmonics (HH) $K^{\pm}_{[\lambda]}$, we now expand the wave function $\psi(R,\Omega)$ in the following form:

$$\psi^{\pm}(R,\Omega)=\sum_{[\lambda']}\phi_{[\lambda']}(R)K^{\pm}_{[\lambda']}(\Omega) \tag{8}$$

where $\phi_{[\lambda']}(R)$ is the radial part, which is determined by the second-order differential equation

$$\left[-\frac{1}{2}\left(\frac{\mathrm{d}^2}{\mathrm{d}R^2}+\frac{3}{R}\frac{\mathrm{d}}{\mathrm{d}R}-\frac{\lambda(\lambda+2)}{R^2}\right)+\frac{1}{8}\gamma^2R^2+\frac{1}{2}\gamma(l_1+l_2)-E\right]$$
$$\times\phi_{[\lambda]}(R)\delta_{[\lambda],[\lambda']}=-\frac{1}{R}\sum_{[\lambda']}U_{[\lambda][\lambda']}\phi_{[\lambda']}(R) \tag{9}$$

where $\delta_{[\lambda],[\lambda']}=\delta_{\nu,\nu'}\delta_{l_1,l_1'}\delta_{l_2,l_2'}$, and $U_{[\lambda][\lambda']}$ is the coupling constant [11, 12]

$$U_{[\lambda][\lambda']}=\int K^{*\pm}_{[\lambda]}(\Omega^{(c)})U(\Omega^{(c)})K^{\pm}_{[\lambda']}(\Omega^{(c)})\mathrm{d}\Omega^{(c)}.$$

In the absence of magnetic field, from equation (9) we can find that $\phi_{[\lambda]}(R)\to \mathrm{e}^{-\sqrt{-2E}R}$ as $R\to\infty$. Therefore, in the low magnetic field, the eigenfunction should contain an exponential form,

$$\phi_{[\lambda]}(R)=u_{[\lambda]}(R)\mathrm{e}^{-\frac{R}{2}}. \tag{10}$$

In addition, the Coulomb energy between particles dominates over the energy of the low magnetic field. In this case, the wave function of $\phi_{[\lambda]}(R)$ or $u_{[\lambda]}(R)$ can be expanded in terms of the generalized Laguerre polynomials (GLP) $L^{(\alpha)}_n(R)$, i.e. the eigenfunctions of a single-electron in a two-dimensional Coulombic potential, which is a fast convergence basis,

$$u_{[\lambda]}(R)=\sum_{n'=0}^{\infty}\sqrt{\frac{n'!}{\Gamma(n'+4)}}C_{n'[\lambda]}L^{(\alpha)}_{n'}(R) \tag{11}$$

Substituting equation (11) into equation (9) with $\alpha=3$, multiplying the equation by $R^3\sqrt{n!/\Gamma(n+4)}L^{(3)}_n(R)\mathrm{e}^{-R/2}$, and integrating over R, we obtain

$$\sum_{n'=0}^{\infty}C_{n'[\lambda]}\sqrt{\frac{n'!n!}{\Gamma(n'+4)\Gamma(n+4)}}\{-\left(\frac{3}{2}+n'\right)G_{n,n'}(2,3,3)-[n'+\lambda(\lambda+2)]G_{n,n'}(1,3,3)$$
$$+(n'+3)G_{n,n'-1}(1,3,3)-\frac{1}{4}\gamma^2G_{n,n'}(5,3,3)\}$$
$$-\sum_{n'[\lambda']}2U_{[\lambda][\lambda']}C_{n'[\lambda']}\sqrt{\frac{n'!n!}{\Gamma(n'+4)\Gamma(n+4)}}G_{n,n'}(2,3,3)$$
$$=-E_pC_{n'[\lambda']}\delta_{[\lambda],[\lambda']}\delta_{n,n'} \tag{12}$$

where $E_p=1/4-\gamma M+2E$, $M=l_1+l_2$ (which is the total angular momentum component in the z direction), E is the eigenenergy of a D^- centre, and

$$G_{n,n'}(\lambda,\mu,\mu')=\int_0^{\infty}R^{\lambda}L^{\mu}_n(R)L^{\mu'}_{n'}(R)\mathrm{e}^{-R}\mathrm{d}R$$
$$=(-1)^{n+n'}\Gamma(\lambda+1)\sum_k\begin{pmatrix}\lambda-\mu\\ n-k\end{pmatrix}\begin{pmatrix}\lambda-\mu'\\ n'-k\end{pmatrix}\begin{pmatrix}\lambda+k\\ k\end{pmatrix}.$$

Equation (12) is a group of linear equations with coefficients$\{C_{n',[\lambda']}\}$, from which we can obtain the eigenvalue E_p.

In a high magnetic field, to obtain fast convergence of equation (9), we set $\rho = \gamma^{1/2}R$ and then expand the wave function as the linear combination of products of pairs of one-electron ground-state eigenfunctions in the high magnetic field limit, as follows:

$$\phi_{[\lambda]}(\rho) = \kappa_{[\lambda]}(\rho)\mathrm{e}^{-\frac{1}{4}\rho^2}\rho^{\lambda}. \tag{13}$$

Given $\rho^2 = 2\xi$, similar to the low magnetic field, the wave function can be expanded in terms of the generalized Laguerre polynomial as

$$\kappa_{[\lambda]}(\xi) = \sum_{n'} C_{n'[\lambda]}\sqrt{\frac{n'!}{(n'+\lambda+1)!}}L_{n'}^{\lambda+1}(\xi). \tag{14}$$

In the same way as for the low magnetic field, the eigenequation for the high magnetic field is derived to be

$$-\left(n+\frac{\lambda+2}{2}\right)\bar{C}_{n[\lambda]}\delta_{[\lambda][\lambda']}\delta_{nn'} - \sum_{n'[\lambda']} 2^{\frac{\lambda'-\lambda-1}{2}}\sqrt{\frac{n'!n!}{(n'+\lambda'+1)!(n+\lambda+1)!}}$$
$$\frac{U_{[\lambda][\lambda']}}{\gamma^{\frac{1}{2}}}\bar{C}_{n'[\lambda']}G_{n,n'}\left(\frac{\lambda+\lambda'+1}{2},\lambda+1,\lambda'+1\right) = E_s\bar{C}_{n'[\lambda']}\delta_{[\lambda][\lambda']}\delta_{nn'} \tag{15}$$

where $E_s = -E/\gamma + M/2$.

The binding energy of D^- is written as:

$$E_b = E_{D^0} + E_e - E_{D^-}$$

where E_e is the ground-state energy of a free electron in a magnetic field, and E_{D^0} is the ground-state energy of D^0 which can be obtained by solving the Schrödinger equation of a two-dimensional donor in a magnetic field [13–16].

3. Numerical results and discussions

The strength of the magnetic field can be denoted by the dimensionless quantity γ, so $\gamma = 2$ means that the magnetic field energy equals the Coulomb energy. To observe the convergence of the ground state energies in the low and high magnetic fields, the energies of $\gamma = 0.2$ and $\gamma = 20$ are listed in tables 1 and 2, respectively. We can see that, with the increase of the total number of basis functions, the convergence is steady, but the effects of the angular and radial parts on the convergence are different. In the case of low magnetic field, the convergence of energy with N_{GLP} (the number of the generalized Laguerre polynomials) is faster than that in high fields, but this is not the case with N_{HH} (the number of the hyperspherical harmonics). Hence a large number of hyperspherical harmonics in low fields and more Laguerre polynomials in high fields are needed. On the other hand, a small number of N_{GLP} in the low fields and a small number of N_{HH} in the high fields are sufficient to obtain accurate eigenvalues. We have calculated the energy for $\gamma = 2$ with equations (12) and (15), respectively, and found that 171 N_{HH} and 30 N_{GLP} with equation (12) but 78 N_{HH} and 40 N_{GLP} with equation (15) were needed to obtain a value with a difference smaller than 10^{-3}.

With coordinates (a) and (c) in equation (5), nearly the same results were obtained for the ground state. However, for the excited states, the results of the space antisymmetric states (spin triplet states) with set (a) are higher than those with set (c). The reason is that the electron exchange energy of the space antisymmetric states is attractive, which makes the total energy decrease. In the following, all the calculations were performed with set (c).

To our knowledge this is the first study of the effect of magnetic field on the ground state energy of a two-dimensional D^- centre. In figures 2(a) and 2(b), we can see that the present

results agree well with those of the 22-term variational calculation [5] in a low magnetic field and the results of [3] in a high magnetic field. Moreover, if the wave function is expanded in more bases, more subtle results can be achieved. In a high magnetic field, our results are better than those obtained in the high magnetic field limit of [3]. In the high magnetic field limit, the Coulomb energy is considered to be zero, and the two electrons are presumed to be in the lowest Landau levels. In fact, this is not the case in a finite field. The electrons can be located in other Landau levels because of the existence of Coulomb terms. Our calculations include the other Landau levels, which makes the energies lower than in [3]. The higher the magnetic fields are, the more the electrons are located in the lowest Landau levels, so that our results tend to approach those of [3] in the high magnetic field limit.

In order to compare the ground-state energy of our calculation with that obtained by experiments performed with far-infrared magnetotransmission and magnetophotoconductivity measurements [2], we present in figure 3 the binding energies of our results and the experimental values of [2]. It can be seen that the slope variation of our theoretical binding energy as a function of magnetic field is steeper than that of experimental results. For comparison, the theoretical binding energy of 3D D^- centres [17, 18] is also plotted and this is reasonably in agreement with experiment [19]. We can see that the curve of the experimental binding energy is much steeper than that of 3D D^- centres. The experiments in [2] were performed in 100 Å well and 100 Å barrier GaAs/GaAlAs MQW samples, which can be considered

Table 1. Ground-state energies of D^- for $\gamma = 0.2$ obtained by diagonalizing equation (12). N_{GLP} is the number of generalized Laguerre polynomials and N_{HH} the number of hyperspherical harmonics.

	N_{GLP}				
N_{HH}	10	15	20	25	30
21	−2.03813660	−2.04485924	−2.04509226	−2.04511902	−2.04512423
28	−2.09333212	−2.09952808	−2.09975414	−2.09978017	−2.09978523
36	−2.09345002	−2.09964818	−2.09987444	−2.09990053	−2.09990561
45	−2.12050694	−2.12639523	−2.12661701	−2.12664241	−2.12664734
55	−2.12057532	−2.12646477	−2.12668664	−2.12671207	−2.12671701
66	−2.13842271	−2.14413028	−2.14434970	−2.14437468	−2.14437952
78	−2.13845469	−2.14416276	−2.14438222	−2.14440721	−2.14441206
91	−2.14933979	−2.15492528	−2.15514274	−2.15516736	−2.15517213
105	−2.14936143	−2.15494726	−2.15516474	−2.15518937	−2.15519415
120	−2.15732790	−2.16283130	−2.16304731	−2.16306243	−2.16306872

Table 2. Ground-state energies of D^- for $\gamma = 20$ obtained by diagonalizing equation (15). The definitions of N_{GLP} and N_{HH} are as above.

	N_{GLP}					
N_{HH}	10	15	20	25	30	35
15	11.8643091	11.8238540	11.8039144	11.7924361	11.7852518	11.7803882
21	11.8640795	11.8234929	11.8034485	11.7919239	11.7846413	11.7797278
28	11.8592906	11.8153281	11.7924274	11.7786040	11.7694903	11.7631125
36	11.8592688	11.8152810	11.7923530	11.7786036	11.7694710	11.7629674
45	11.8587363	11.8140867	11.7904201	11.7758529	11.7660596	11.7529078
55	11.8587335	11.8140644	11.7904052	11.7758397	11.7660276	11.7590376
66	11.8587235	11.8138201	11.7898919	11.7750132	11.7648871	11.7575717
78	11.8586483	11.8138176	11.7898894	11.7750087	11.7648799	11.7575617

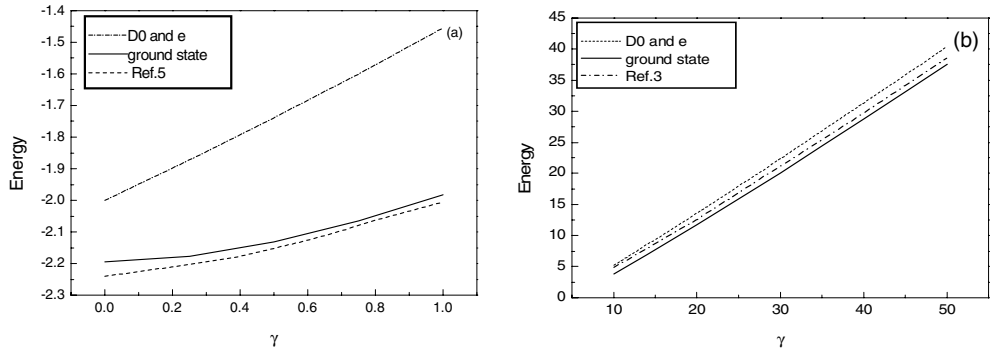

Figure 2. (a) Ground state energies of D^- centre as a function of magnetic field. The solid curve is our result obtained from diagonalizing equation (12). The dashed curve is obtained with the 22-term variational method [5]. The ground state energy of the D^0 centre and the free electron energy are also plotted (dash-dotted curve). (b) Ground state energies of D^- centre in high magnetic fields. The solid curve is our result obtained from diagonalizing equation (15). The dash-dotted curve was obtained by Larsen using the high magnetic field limit model [3]. The energy sum of the D^0 centre and the free electron is also plotted (dashed curve)

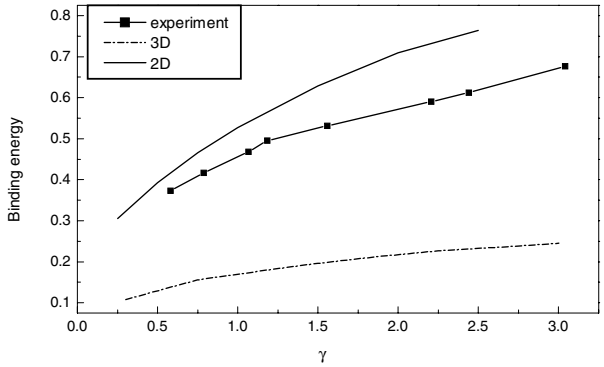

Figure 3. Ground state energies of D^- centre as a function of magnetic field. The solid curve is our result. The experimental data (square-dot curve) are from [2]. For comparison, the theoretical binding energy of the 3D D^- centre is shown (dash-dotted curve [18])

as having a quasi-2D structure, while our results were calculated for a strict 2D structure. Theoretically, the quasi-2D structure or the quantum well structure is between the strictly 2D and 3D structures; its physical properties such as energy levels should lie between the 2D and 3D properties. Furthermore, the potential produced by donor atoms and polarized by the Coulomb field of distant electrons in a 3D centre is completely different from that of the 2D case, which accounts for the difference of the charge distribution. For the 2D and quasi-2D centres the charge distribution of the donor electron is not spherically symmetric as in three dimensions but, instead, is oblate [3]. This produces a dramatic deepening of the ground state [2] of the 2D D^- centre, so our results are reasonable for a strict 2D structure.

With equation (15) we have calculated the binding energies E_{D^-} of several low excitation states such as $M = -1, -2, -3$ and -4, and then calculated their symmetric and antisymmetric states, respectively, in set (c). For the same value of M, the energies of symmetric states are higher than those of antisymmetric states, and all symmetric states are unbound. We find that, with the increase of the absolute value of M the energy differences $E^+(M) - E^-(M)$

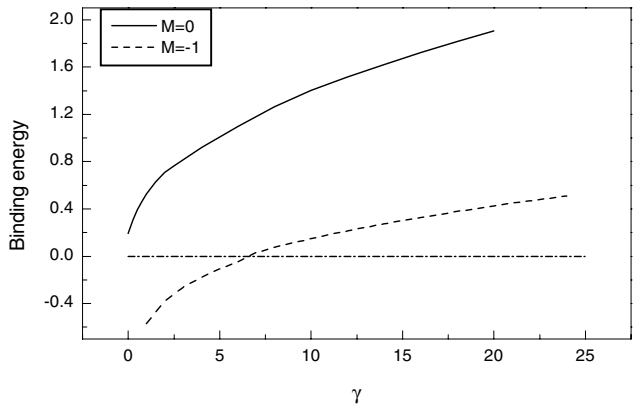

Figure 4. Binding energies as a function of magnetic fields for $M = 0$, symmetric state (solid curve) and $M = -1$, antisymmetric state (dashed curve). The zero value is also plotted (dash-dotted curve).

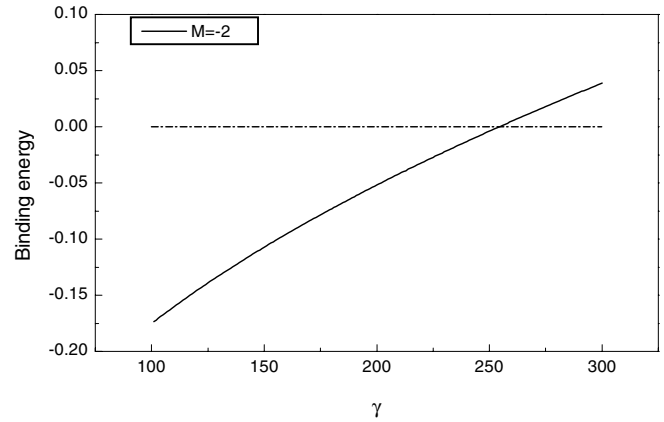

Figure 5. Binding energy as a function of magnetic field for $M = -2$, antisymmetric state (solid curve). The dash-dotted curve indicates the binding energy zero value.

quickly become smaller, which is in agreement with the results in the high magnetic field limit [3]. In this limit, there exist four bound states. Since at $B = 0$ only the ground state is a bound state, there must exist thresholds of the magnetic field at which some excited D^- states become bound and remain so in a higher field. In figures 4, 5 and 6, the binding energies of the ground and several low excited antisymmetric states are presented. It can be seen that binding occurs for the lowest-lying $M = 0$ symmetric state and $M = -1$, -2 and -3 antisymmetric states. With the increase of the absolute value of M, the variation of the binding energies with magnetic field becomes less steep, and the critical magnetic field at which the states change from unbound to bound becomes larger. For the $M = -1$ antisymmetric state, the critical value is $\gamma_{c1} \approx 7$ (about 46.6 T), which is very high for current laboratory conditions. For the $M = -2$ and $M = -3$ antisymmetric states, the critical values are $\gamma_{c2} \approx 255$ and $\gamma_{c3} \approx 10^{5.8}$, respectively, which is too high to be reached in the laboratory. That is to say, there may exist only one or two bound states under present experimental conditions. Moreover, the optical transition between the two bound states is not possible according to the dipole selection rule

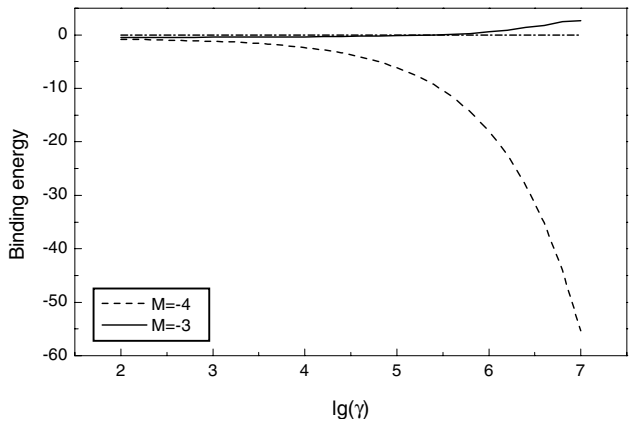

Figure 6. Binding energies as a function of $\lg(\gamma)$ of antisymmetric states, for $M = -3$ (solid curve) and $M = -4$ (dashed curve). The dash-dotted line indicates the binding energy zero value.

($\Delta M = \pm 1$) [3] and the general selection rule that requires the initial and final space states to be either both symmetric or both antisymmetric. Hence there is no discrete optical transition spectrum observable from 2D D^- centres. The two-dimensional D^- model gives us some quantitative information on the behaviour of D^- centres in the middle of quantum wells. If the electrons are allowed to move along the growth axis under confinement of the narrow quantum-well potential, the magnetic field can produce an attractive quadrupole potential far from the donor centre [3], which may bind more states in a finite field. Further investigation will be conducted on D^- centres in quantum wells and superlattice structures in various materials.

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